## Math 2270, Midterm 1

PRINT YOUR NAME:

| Question | Points | Score |
| :---: | :---: | :---: |
| 1 | 16 |  |
| 2 | 12 |  |
| 3 | 8 |  |
| 4 | 9 |  |
| 5 | 15 |  |
| 6 | 20 |  |
| 7 | 12 |  |
| 8 | 8 |  |
| Total: | 100 |  |

- No advanced calculators or cell phones or other electronic devices allowed at any time.
- Show all your reasoning and work for full credit, except where otherwise indicated.
- You have 50 minutes and the exam is 100 points.
- You do not need to simplify numerical expressions. For example leave fractions like $100 / 7$ or expressions like $\ln (3) / 2$ as is.
- Do the best you can!

1. (16 points) Find the general solution to the below linear system of equations if they are consistent. Write answer in parametric form. If not, write that there is no solution.

$$
\begin{aligned}
x_{1}-x_{2}-3 x_{3} & =5 \\
-2 x_{1}+4 x_{2}+8 x_{3} & =-14 \\
x_{1}-4 x_{2}-6 x_{3} & =11
\end{aligned}
$$

$$
\vec{x}=
$$

2. (12 points) Is the vector $\vec{v}$ in the span of the other vectors $\vec{u}_{1}, \vec{u}_{2}, \vec{u}_{3}$ ?

$$
\vec{u}_{1}=\left[\begin{array}{c}
1 \\
0 \\
-1
\end{array}\right], \vec{u}_{2}=\left[\begin{array}{c}
-2 \\
1 \\
3
\end{array}\right], \overrightarrow{u_{3}}=\left[\begin{array}{c}
-4 \\
1 \\
5
\end{array}\right], \quad \vec{v}=\left[\begin{array}{c}
7 \\
-2 \\
-5
\end{array}\right]
$$

3. (8 points) Which of the following sets of vectors are bases for $\mathbb{R}^{2}$ ? Circle either "Basis" or "Not basis" to the left of each set of vectors according to your answer.

Basis Not basis

$$
\left\{\left[\begin{array}{c}
3 \\
-6
\end{array}\right],\left[\begin{array}{c}
6 \\
-12
\end{array}\right],\left[\begin{array}{c}
9 \\
-13
\end{array}\right]\right\}
$$

Basis Not basis

$$
\left\{\left[\begin{array}{l}
0 \\
0
\end{array}\right],\left[\begin{array}{l}
0 \\
1
\end{array}\right]\right\}
$$

Basis Not basis

$$
\left\{\left[\begin{array}{l}
1 \\
1
\end{array}\right]\right\}
$$

Basis Not basis

$$
\left\{\left[\begin{array}{c}
-2 \\
1
\end{array}\right],\left[\begin{array}{c}
0 \\
-1
\end{array}\right]\right\}
$$

4. (9 points) Determine whether each map is one-to-one, onto, both (isomorphism), or neither. Circle the appropriate answer, circling both if both are true and neither if neither are true.
one-to-one onto Rotation 120 degrees counterclockwise.

$$
A=\left[\begin{array}{ccc}
1 & 0 & 2 \\
12 & 1 & 20 \\
11 & -3 & 18
\end{array}\right] \quad B=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right]
$$

$$
A=\left[\begin{array}{ccc}
1 & -2 & -2 \\
0 & 1 & 1
\end{array}\right]
$$

5. (15 points) Find determinants of the following matrices if possible or write that the determinant is not defined otherwise. Then figure out whether the matrix is invertible or not and circle the corresponding answer:

Invertible Not Invertible

$$
\left[\begin{array}{cc}
-2 & 5 \\
8 & 13
\end{array}\right]
$$



Invertible Not Invertible

$$
\left[\begin{array}{cccc}
-13 & 2 & 4 & 5 \\
4 & 2 & -3 & 7 \\
1 & 1 & -4 & 9
\end{array}\right]
$$

$$
\operatorname{det}=
$$

$\qquad$
Invertible Not Invertible

$$
\left[\begin{array}{ccc}
-2 & 1 & 0 \\
3 & 20 & -4 \\
10 & -5 & 1
\end{array}\right]
$$

$\operatorname{det}=$
6. (20 points) Find a basis for the null space $\operatorname{Nul} A$ and column space $\operatorname{Col} A$ of the matrix $A$. Determine the dimensions of the null space and column space.

$$
A=\left[\begin{array}{cccc}
1 & 1 & 3 & 5 \\
2 & -2 & -6 & 2 \\
0 & 2 & 6 & 7
\end{array}\right]
$$

Null Space Nul $A$ :
$\operatorname{Nul} A$ basis:
$\operatorname{dim} \operatorname{Nul} A:$

Column Space $\operatorname{Col} A$ :
$\operatorname{Col} A$ basis:
$\operatorname{dim} \operatorname{Col} A:$
7. (12 points) Find the inverse and determinant of the matrix

$$
A=\left[\begin{array}{ll}
-3 & 5 \\
-7 & 9
\end{array}\right]
$$


8. (8 points)

1. Is the function $F: \mathbb{R}^{2} \rightarrow \mathbb{R}^{2}$ defined by

$$
F\left(\left[\begin{array}{l}
a \\
b
\end{array}\right]\right)=\left[\begin{array}{c}
1-a b \\
a+b
\end{array}\right]
$$

linear or not? Circle your answer.
2. Let $F: \mathbb{R}^{3} \rightarrow \mathbb{R}$ be the linear transformation

$$
F\left(\left[\begin{array}{l}
a \\
b \\
c
\end{array}\right]\right)=3 a+2 b+c
$$

If possible, find a matrix $A$ such that $F(\vec{x})=A \vec{x}$. If not, write that $A$ does not exist.

$$
A=
$$

$\qquad$

